$$V(X)=\int_\alpha^\beta (x-m)^2 f(x)\,dx$$
$$=\int_\alpha^\beta x^2 f(x)\,dx-m^2 \qquad E(X)=\int_\alpha^\beta xf(x)\,dx$$
$$\Leftarrow m=E(X)$$

$$V(aX+b)=\int_\alpha^\beta \left[ax+b-\left\{aE(X)+b\right\}\right]^2 f(x)\,dx$$
$$=a^2\int_\alpha^\beta \left\{x-E(X)^2\right\} f(x)\,dx$$
$$=a^2V(X)$$

$$\int_a^b f(x)\,dx=\lim_{n\to\infty}\sum_{k=0}^{n-1} f(x_k)\cdot\Delta x$$
$$=\lim_{n\to\infty} f(x_{k-1})\cdot\Delta x$$

$$\lim_{n\to\infty}\sum_{k=1}^{n} f\left(a+\frac{(b-a)k}{n}\cdot\frac{b-a}{n}\right)$$

$$V(aX+b)=\int_\alpha^\beta \left[ax+b-\left\{aE(X)+b\right\}\right]^2 f(x)\,dx$$
$$=a^2\int_\alpha^\beta \left\{x-E(X)^2\right\} f(x)\,dx=a^2V(X)$$

$$\sigma(X)=\sqrt{\int_\alpha^\beta (x-m)^2 f(x)\,dx} \qquad \sigma^2=\frac{\sum (x_i-m)f_i}{\sum f_i}$$
$$=\sqrt{\int_\alpha^\beta x^2 f(x)\,dx-m^2} \qquad =\frac{\sum x_i f_i}{\sum f_i}-m^2$$

$$\frac{-b\pm\sqrt{b^2-4ac}}{2a} \qquad d=\frac{\left|ax_1+by_1+c\right|}{\sqrt{a^2+b^2}}$$

$$0.21\times 1\frac{2}{7}\div 0.9=\frac{21}{100}\times\frac{9}{7}\div\frac{9}{10}$$
$$=\frac{21}{100}\times\frac{9}{7}\times\frac{10}{9}$$
$$=\frac{\overset{3}{\cancel{21}}}{\underset{10}{\cancel{100}}}\times\frac{\overset{1}{\cancel{9}}}{\underset{1}{\cancel{7}}}\times\frac{\overset{1}{\cancel{10}}}{\underset{1}{\cancel{9}}}$$
$$=\frac{3}{10}$$

$$\frac{\overset{3}{\cancel{21}}}{\underset{10}{\cancel{100}}}\times\frac{\overset{1}{\cancel{9}}}{\underset{1}{\cancel{7}}}\times\frac{\overset{1}{\cancel{10}}}{\underset{1}{\cancel{9}}}$$

$$\int_{-\infty}^{+\infty}\left|a(t)\right|^2 dx\cdot\left|\lambda-\frac{\int_{-\infty}^{+\infty}A(t)B(t)}{\int_{-\infty}^{+\infty}\left|B(t)\right|^2 dt}\right|^2$$

$$\overline{C}_{mn}(\tau)=\lim_{T\to\infty}\left[\frac{1}{T}\int_{-T/2}^{+T/2} g_m(t)\,g_n(t-\tau)\,dt\right]$$

$$P_o(\omega)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h(\xi_1)\,h(\xi_2)\times\overline{C}_i(\zeta)\exp\left\{+j\omega(\zeta-\xi_1+\xi_2)\right\}d\zeta\, d\xi_1\, d\xi_2$$

$$\left[e_s\right]=\sqrt{\int_{-\infty}^{+\infty}\sigma^2\frac{\exp\left(-x^2/2\sigma^2\right)}{\sqrt{2\pi}\,\sigma}\,dx}=\sigma \qquad h(t_n)=\sum_{m=1}^{M}\left(\left.\frac{d\left[z^{1-m}H(z)^{-1}\right]}{dz}\right|_{z=z_m}\right)^{-1}$$

$$\overbrace{\text{Moles of AgBr}}^{\text{n}}=\overbrace{\left(56.4\text{g}\,\cancel{\text{AgBr}}\right)\times\left(\frac{1\,\text{mol AgBr}}{187.78\,\text{g}\,\cancel{\text{AgBr}}}\right)}^{\text{n/M}}$$

$$\underbrace{\text{Ba(NO}_3)_2\text{(aq)}}_{\text{solution}}+\underbrace{2\text{NH}_4\text{IO}_3\text{(aq)}}_{\text{solution}}\rightarrow\underbrace{\text{Ba(IO}_3)\text{(s)}}_{\text{pre}}+\underbrace{2\text{NH}_4\text{NO}_3\text{(aq)}}_{\text{solution}}$$

$$Ba^{2+}(aq)+2NO_3^-(aq)+2NH_4^+(aq)+2IO_3^-(aq)\rightarrow Ba(IO_3)_2(s)+2NH_4^+(aq)+2NO_3^-(aq)$$

$$\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\infty}\sqrt{Test}}}}}}}}}}}}} \quad \begin{pmatrix}\frac{1}{\frac{2}{\frac{3}{4}}} & \frac{1}{\frac{2}{3}}\end{pmatrix} \qquad \sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{unlimited}\sqrt{Test2}}}}}}}}}}}} \quad \begin{pmatrix}\frac{1}{\frac{2}{\frac{3}{4}}} & \frac{1}{\frac{2}{3}}\end{pmatrix} \xleftrightarrow{\text{other way}} \xleftarrow{\text{this}}$$

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$$\begin{array}{r}123\\23\overline{)68909}\end{array} \quad \boxed{\begin{array}{r}123\\23\overline{)68909}\end{array}} \quad \begin{array}{r}34\overline{)567}\\\underline{}89\end{array}$$

1.2 구좌표계 (r,θ,φ)에 대한 미분연산

$$(\nabla\cdot v)\ =\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\upsilon_r)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\upsilon_\theta\sin\theta)+\frac{1}{r\sin\theta}\frac{\partial\upsilon_\varphi}{\partial\varphi}$$

$$(\nabla^2 s)\ =\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial s}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 s}{\partial\varphi^2}$$

$$(\tau:\nabla v)=\tau\left(\frac{\partial\upsilon}{\partial r}\right)+\tau_{r\theta}\left(\frac{1}{r}\frac{\partial\upsilon_r}{\partial\theta}-\frac{\upsilon_\theta}{r}\right)+\tau_{r\varphi}\left(\frac{1}{r\sin\theta}\frac{\partial\upsilon_r}{\partial\varphi}-\frac{\upsilon_\varphi}{r}\right)$$
$$+\tau_{\theta r}\left(\frac{\partial\upsilon_r}{\partial r}\right)+\tau_{\theta\theta}\left(\frac{1}{r}\frac{\partial\upsilon_\theta}{\partial\theta}-\frac{\upsilon_\theta}{r}\right)+\tau_{r\varphi}\left(\frac{1}{r\sin\theta}\frac{\partial\upsilon_\theta}{\partial\varphi}-\frac{\upsilon_\varphi}{r}\cot\theta\right)$$

$$(\nabla\cdot\tau)_r=\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{rr})+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\tau_{\theta r}\sin\theta)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}\tau_{\varphi r}-\frac{\tau_{\theta\theta}+\tau_{\varphi\varphi}}{r}$$

$$(\nabla\cdot\tau)_\theta=\frac{1}{r^3}\frac{\partial}{\partial r}(r^2\tau_{r\theta})+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\tau_{\theta\theta}\sin\theta)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}\tau_{\varphi\theta}-\frac{(\tau_{\theta r}-\tau_{r\theta})-\tau_{\varphi\varphi}\cot\theta}{r}$$

$$(\nabla^2 v)_r\ =\frac{\partial}{\partial r}\left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r)\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial v_r}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_r}{\partial\varphi^2}-\frac{2}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\upsilon_\theta\sin\theta)-\frac{2}{r^2\sin\theta}\frac{\partial\upsilon_\varphi}{\partial\varphi}$$

8.18 컴보기응력과 와점성계수

식 (8.18.1)을 식 (8.17.16)과 비교하여 이 관계를 직교좌표계에 대하여 표시하면

$$[\bar\tau]_{lam}=(\bar\tau_{ij})\,lam=\begin{bmatrix}\bar\tau_{xx}&\bar\tau_{xy}&\bar\tau_{xz}\\ \bar\tau_{yx}&\bar\tau_{yy}&\bar\tau_{yz}\\ \bar\tau_{zx}&\bar\tau_{zy}&\bar\tau_{zz}\end{bmatrix}_{lam}=\begin{bmatrix}2\mu\frac{\partial\bar u}{\partial x}&\mu\left(\frac{\partial\bar u}{\partial y}+\frac{\partial\bar\upsilon}{\partial\chi}\right)&\mu\left(\frac{\partial\bar u}{\partial z}+\frac{\partial\bar w}{\partial\chi}\right)\\ \mu\left(\frac{\partial\bar\upsilon}{\partial x}+\frac{\partial\bar u}{\partial y}\right)&2\mu\frac{\partial\bar\upsilon}{\partial y}&\mu\left(\frac{\partial\bar\upsilon}{\partial z}+\frac{\partial\bar w}{\partial y}\right)\\ \mu\left(\frac{\partial\bar w}{\partial x}+\frac{\partial\bar w}{\partial z}\right)&\mu\left(\frac{\partial\bar w}{\partial y}+\frac{\partial\bar\upsilon}{\partial z}\right)&2\mu\frac{\partial\bar w}{\partial z}\end{bmatrix}\qquad(8.18.4)$$

8.23 벽전단응력 τ_w와 조도 e의 계산

$\zeta=y/R$라고 놓으면 $dy=Rd\zeta$ 이므로

$$(\bar u_{\max}-V)\pi R^2=\frac{2\pi R^2\upsilon*}{\varkappa}\left[\int_R^0\zeta\ln\zeta-\int_R^0\ln\zeta d\zeta\right]$$
$$=\frac{2\pi R^2\upsilon*}{\varkappa}\left[\left(\frac{\zeta^2}{2}\ln\zeta-\frac{\zeta^2}{2}\right)-(\zeta\ln\zeta-\zeta)\right]_0^1$$

기본문제 11-9. 다음 두 식을 만족하는 다항 식 $f(x)$를 구하여라.

$$\lim_{x\to\infty}\frac{f(x)}{2x^2+x+1},\qquad\lim_{x\to 2}\frac{f(x)}{x^2-x-2}=1$$

$$\lim_{x\to 2}\frac{f(x)}{(x-2)(x+1)}=\lim_{x\to 2}\frac{2x^2+bx-8-2b}{(x-2)(x+1)}$$
$$=\lim_{x\to 2}\frac{(x-2)(2x+b+4)}{(x-2)(x+1)}=\frac{b+3}{3}=1$$

$\therefore b=-5\qquad\therefore c=10-8=2\qquad\therefore \boldsymbol{f(x)=2x^2-5x+2}$

$$\underbrace{2NH_4IO_3(aq)}_{\text{solution}}$$

$$\begin{array}{rrr} & 2 & 3\\ \times & 3 & 1\\ \hline & 2 & 3\\ 6 & 9 & \\ \hline 7 & 1 & 3\end{array}$$

27. $f(x)=a^x(a>0)$일 때, 다음 중에서 옳지 않은 것은?

① $f(x)f(y)=f(x+y)$　　② $f(x)\div f(y)=f(x-y)$

③ $\{f(x)\}^y=f(xy)$　　④ $f(x\div y)=f(x)-f(y)$

$$x+1\overline{\big)\,x^2+3x+2}\qquad x+3$$

1. Random Numbers, Variates, and Stochastic Process Generation

1. Apply the inverse-transform method to variate generation from a Laplace distribution (shifted two-sided exponential distribution)

$$f(x) = \frac{1}{2\beta} \exp\left[\frac{-|x-\theta|}{\beta}\right], \quad -\infty < x < \infty, \quad \beta > 0.$$

2. Apply the inverse-transform method to variate generation from the extreme value distribution

$$f(x) = \frac{1}{\sigma} \exp\left[-\frac{1}{\sigma}(x-\mu) - \exp\left(\frac{-(x-\mu)}{\sigma}\right)\right], \quad -\infty < x < \infty.$$

3. Consider the triangular variate, with the pdf

$$f(x) = \begin{cases} 0 & \text{if } x < 2a \text{ } or \text{ } x \geq 2b \\ \dfrac{x-2a}{(b-a)^2} & \text{if } 2a \leq x < a+b \\ \dfrac{2b-x}{(b-a)^2} & \text{if } a+b \leq x < 2b, \end{cases} \text{, and cdf } \quad F(x) = \begin{cases} 0 & \text{if } x < 2a \\ \dfrac{x-2a}{(b-a)} & \text{if } 2a \leq x < a+b \\ 1 - \dfrac{(2b-x)^2}{2(b-a)^2} & \text{if } a+b \leq x < 2b \\ 1 & \text{if } x \geq 2b. \end{cases}$$

Show that applying the inverse-transform method yields

$$X = \begin{cases} 2a + (b-a)\sqrt{2U} & \text{if } 0 \leq U < 0.5 \\ 2b + (a-b)\sqrt{2(1-U)} & \text{if } 0.5 \leq U < 1. \end{cases}$$

4. Let

$$f(x) = \begin{cases} C_i x & x_{i-1} \leq x < x_i, i = 1, \ldots, n \\ 0 & \text{otherwise} \end{cases}$$

$$x_0 = a, \quad x_n = b, \quad C_i \geq 0, \quad a \geq 0.$$

Using theinverse-transform method, show that

$$X = \left[x_{i-1}^2 + \frac{2(U - F_{i-1})}{C_i}\right]^{1/2}$$

where $F_i = \sum_{j=1}^{i} \int_{x_{j-1}}^{x_j} C_j x \, dx$. Describe an algorithm for variate generation from $f(x)$.

5. Apply the inverse-transform method to generate a variate from the following pdf: $f(x) = \begin{cases} \dfrac{1}{n+1} & x = 0, 1, \ldots, n \\ 0 & \text{otherwise} \end{cases}$

6. Devise an acceptance-rejection algorithm for generating a variate from the pdf: $f(x) = \sqrt{\dfrac{2}{\pi}} e^{-x^2/2}, \quad x \geq 0,$

using the representation $f(x) = Ch(x, \beta) g(x)$, where

$$h(x, \beta) = \frac{1}{\beta} e^{-x/\beta}, \quad x > 0, \beta > 0, \text{ for fixed } \beta.$$

$$g^*(x) = |Q(x)| \Big/ \int |Q(x)| \, dx, \quad f_k(y_k, \mu_k, \sigma_k) = \frac{1}{(2\pi)^{1/2} \sigma_k} \exp\left(\frac{-(y_k - \mu_k)^2}{2\sigma_k^2}\right). \quad \theta = \int \left(\begin{matrix} \sum_{i=1}^{n} a_i^2 L_i^2(x) \\ \sum_{i=1}^{n} b_i^2 L_i^2(x) \\ \sum_{i=1}^{n} c_i^2 L_i^2(x) \end{matrix} \right)^{1/2} dx$$