Eqs.(6.7). Moreover, our attention as engineers is toward the average or mean value of velocity, pressure, shear stress, etc., in a high-Reynolds-number (turbulent) flow. This approach led Osborne Reynolds in 1895 to rewrite Eqs. (6.7) in terms of mean or time-averaged turbulent variables.

The time mean $\overline{u}$ of a turbulent function *u(x,y,z,t)* is defined by

$$\overline{u}=\frac{1}{T}\int_0^T u\,dt \qquad (6.8)$$

where T is an averaging period taken to be longer than any significant period of the fluctuations themselves. The mean values of turbulent velocity and pressure are illustrated in Fig. 6.7. For turbulent gas and water flows an averaging period $T\approx 5$ s is usually quite adequate.

The fluctuation u' is defined as the deviation of u from its average value

$$u'=u-\overline{u} \qquad (6.9)$$

as shown also in Fig. 6.7. It follows by definition that a fluctuation has zero mean value

$$\overline{u'}=\frac{1}{T}\int_0^T (u-\overline{u})\,dt=\overline{u}-\overline{u}=0 \qquad (6.10)$$

However, the mean square of a fluctuation is not zero and is a measure of the *intensity* of the turbulence

$$\overline{u'^2}=\frac{1}{T}\int_0^T u'^2\,dt \neq 0 \qquad (6.11)$$

Nor in general are the mean fluctuation products such as $\overline{u'v'}$ and $\overline{u'p'}$ zero in a typical tubulent flow.

Reynolds' idea was to split each property into mean plus fluctuating variables

$$u=\overline{u}+u' \qquad v=\overline{v}+v' \qquad w=\overline{w}+w' \qquad p=\overline{p}+p' \qquad (6$$

Substitute these into Eqs. (6.7) and take the time mean of each equation. The continuity relation reduces to

$$\frac{\partial\overline{u}}{\partial x}+\frac{\partial\overline{v}}{\partial y}+\frac{\partial\overline{w}}{\partial z}=0 \qquad (6.13)$$

which is no different from a laminar continuity relation.

However, each component of the momentum equation (6.7*b*), after time averaging, will contain mean values plus three mean products, or *correlations*, of fluctuating velocities. The most important of these is the momentum relation in the mainstream, or *x*, direation, which takes the form

$$\rho\frac{d\overline{u}}{dt}=-\frac{\partial\overline{p}}{\partial x}+\rho g_x+\frac{\partial}{\partial x}\left(\mu\frac{\partial\overline{u}}{\partial x}-\rho\overline{u'^2}\right) +\frac{\partial}{\partial y}\left(\mu\frac{\partial\overline{u}}{\partial y}-\rho\overline{u'v'}\right)+\frac{\partial}{\partial z}\left(\mu\frac{\partial\overline{u}}{\partial z}-\rho\overline{u'w'}\right) \qquad (6.14)$$